
BEYOND SQUARE ROOTS

□ **SQUARE ROOTS REVIEW**

The number 9 has two **square roots**, 3 and -3 . This is because $3^2 = 9$ and $(-3)^2 = 9$. The positive square root of 9 (the 3) is denoted $\sqrt{9}$, and the negative square root of 9 (the -3) is written $-\sqrt{9}$. In other words, $\sqrt{9} = 3$, and only 3, while $-\sqrt{9} = -3$.

In analyzing $\sqrt{-25}$, the “principle” square root of -25 , we discover that we cannot find an answer for this problem, since the square of a real number can never be negative. If there is an answer to $\sqrt{-25}$, it lies outside $\mathbb{R}$, the set of real numbers.

Sometimes the square root radical is written $\sqrt[2]{x}$, with a little 2 cradled in the radical sign. The 2 refers to the fact that taking the square root is essentially the opposite of squaring.

□ **HIGHER ROOTS**

Consider the number 8. Since two cubed is eight, $2^3 = 8$, we can say that 2 is a **cube root** of 8. In fact, it’s the only cube root of 8, simply because there’s no other real number whose cube is 8.

We write $\sqrt[3]{8} = 2$. Perhaps a little surprising is that we can calculate the cube root of a negative number without leaving the real numbers, $\mathbb{R}$. For example, $\sqrt[3]{-27}$ equals -3 , since

$$(-3)^3 = -27$$

$$[-3 \cdot -3 \cdot -3 = -27]$$

The number 16 has two **fourth roots**. The positive fourth root is $\sqrt[4]{16} = 2$, and the negative fourth root is $-\sqrt[4]{16} = -2$. After all, both 2 and -2 raised to the fourth power result in 16. However, just like square roots, $\sqrt[4]{-1}$ is not a real number.

The **fifth root** of 32 is 2; that is, $\sqrt[5]{32} = 2$. This is because $2^5 = 32$. Like cube roots, we can calculate the fifth root of a negative number. For example, $\sqrt[5]{-243}$ equals -3 , since $(-3)^5 = -243$.

Homework

1. Find the square root(s) of
 - a. 100 b. 15 c. 0 d. -36 e. 1
2. Find the cube root(s) of
 - a. 64 b. -125 c. 0 d. 20 e. 1
3. Find the fourth root(s) of
 - a. 81 b. 0 c. -625 d. 25 e. 1
4. Find the fifth root(s) of
 - a. 1 b. 0 c. -243 d. 29 e. 32
5. Evaluate each radical:
 - a. $\sqrt{169}$ b. $\sqrt{225}$ c. $\sqrt[3]{8}$ d. $\sqrt[3]{27}$ e. $\sqrt[3]{-125}$
 - f. $\sqrt[4]{625}$ g. $\sqrt[4]{1}$ h. $\sqrt[4]{-16}$ i. $\sqrt[5]{-32}$ j. $\sqrt[5]{0}$
 - k. $\sqrt[3]{64}$ l. $\sqrt[3]{216}$ m. $\sqrt[3]{-64}$ n. $-\sqrt[5]{-1}$ o. $\sqrt[4]{16}$
 - p. $-\sqrt[4]{81}$ q. $\sqrt[3]{-1}$ r. $-\sqrt[4]{-1}$ s. $\sqrt[5]{243}$ t. $\sqrt{0} + \sqrt[3]{0}$

□ SIMPLIFYING MORE ROOTS

Assume that x and y represent non-negative numbers. Then

$$\sqrt[n]{xy} = \sqrt[n]{x} \sqrt[n]{y}$$

The n th root of a product is the product of the n th roots

Another useful rule for radicals is the following. If x represents a non-negative number ($x \geq 0$), then

$$\sqrt[n]{x^n} = x$$

The n th root cancels the n th power

[The special case of simplifying $\sqrt{x^2}$, where x has negative value, will not be dealt with in this chapter.]

EXAMPLES: Simplify each radical expression:

$$A. \quad \sqrt{200} = \sqrt{100 \times 2} = \sqrt{100} \times \sqrt{2} = 10\sqrt{2}$$

$$B. \quad \sqrt[3]{81} = \sqrt[3]{27 \times 3} = \sqrt[3]{27} \times \sqrt[3]{3} = 3\sqrt[3]{3}$$

$$C. \quad \sqrt[4]{1250} = \sqrt[4]{625 \cdot 2} = \sqrt[4]{625} \cdot \sqrt[4]{2} = 5\sqrt[4]{2}$$

- D. $\sqrt[3]{2250}$ Sometimes the radicand (the 2250) is too big to easily see if there's a perfect cube in it. So let's try a slightly different approach. We factor the 2250 into primes to get

$$2250 = 2 \cdot 3^2 \cdot 5^3$$

Clearly we can take the cube root of 5^3 (it's 5), but there are not enough of the other factors to take their cube roots. So we can write

$$\sqrt[3]{2250} = \sqrt[3]{2 \cdot 3^2 \cdot 5^3} = \sqrt[3]{5^3} \cdot \sqrt[3]{2 \cdot 3^2} = 5\sqrt[3]{18}$$

Homework

6. Simplify each radical:

- | | | | |
|--------------------|---------------------|--------------------|---------------------|
| a. $\sqrt{288}$ | b. $\sqrt[3]{54}$ | c. $\sqrt[3]{16}$ | d. $\sqrt[3]{250}$ |
| e. $\sqrt[4]{32}$ | f. $\sqrt[4]{243}$ | g. $\sqrt[4]{162}$ | h. $\sqrt[4]{1}$ |
| i. $\sqrt[3]{-54}$ | j. $\sqrt[4]{-16}$ | k. $\sqrt[5]{64}$ | l. $\sqrt[5]{486}$ |
| m. $\sqrt[3]{135}$ | n. $\sqrt[4]{162}$ | o. $\sqrt[3]{189}$ | p. $\sqrt[5]{96}$ |
| q. $\sqrt[3]{128}$ | r. $\sqrt[4]{1250}$ | s. $\sqrt[3]{250}$ | t. $\sqrt[3]{432}$ |
| u. $\sqrt[5]{320}$ | v. $\sqrt[3]{48}$ | w. $\sqrt[4]{648}$ | x. $\sqrt[5]{2673}$ |

Review Problems

7. Simplify each radical:

- | | | | |
|--------------------|---------------------|---------------------|---------------------|
| a. $\sqrt[3]{108}$ | b. $\sqrt[4]{405}$ | c. $\sqrt[5]{192}$ | d. $\sqrt[3]{-500}$ |
| e. $\sqrt[4]{-32}$ | f. $\sqrt[5]{-486}$ | g. $\sqrt[3]{3000}$ | h. $\sqrt[4]{567}$ |
| i. $\sqrt[3]{648}$ | j. $\sqrt[5]{320}$ | k. $\sqrt[3]{81}$ | l. $\sqrt[3]{250}$ |
| m. $\sqrt[3]{-56}$ | n. $\sqrt[4]{48}$ | o. $\sqrt[4]{-405}$ | p. $\sqrt[4]{768}$ |
| q. $\sqrt[5]{128}$ | r. $\sqrt[5]{1215}$ | | |

Solutions

- | | | | | | |
|-----------|-------------|-------------------|-------------|----------------------|------------|
| 1. | a. ± 10 | b. $\pm\sqrt{15}$ | c. 0 | d. Not real | e. ± 1 |
| 2. | a. 4 | b. -5 | c. 0 | d. $\sqrt[3]{20}$ | e. 1 |
| 3. | a. ± 3 | b. 0 | c. Not real | d. $\pm\sqrt[4]{25}$ | e. ± 1 |
| 4. | a. 1 | b. 0 | c. -3 | d. $\sqrt[5]{29}$ | e. 2 |
| 5. | a. 13 | b. 15 | c. 2 | d. 3 | e. -5 |
| | f. 5 | g. 1 | h. Not real | i. -2 | j. 0 |
| | k. 4 | l. 6 | m. -4 | n. 1 | o. 2 |
| | p. -3 | q. -1 | r. Not real | s. 3 | t. 0 |

6. a. $12\sqrt{2}$ b. $3\sqrt[3]{2}$ c. $2\sqrt[3]{2}$ d. $5\sqrt[3]{2}$ e. $2\sqrt[4]{2}$ f. $3\sqrt[4]{3}$
 g. $3\sqrt[4]{2}$ h. 1 i. $-3\sqrt[3]{2}$ j. Not real k. $2\sqrt[5]{2}$ l. $3\sqrt[5]{2}$
 m. $3\sqrt[3]{5}$ n. $3\sqrt[4]{2}$ o. $3\sqrt[3]{7}$ p. $2\sqrt[5]{3}$ q. $4\sqrt[3]{2}$ r. $5\sqrt[4]{2}$
 s. $5\sqrt[3]{2}$ t. $6\sqrt[3]{2}$ u. $2\sqrt[5]{10}$ v. $2\sqrt[3]{6}$ w. $3\sqrt[4]{8}$ x. $3\sqrt[5]{11}$
7. a. $3\sqrt[3]{4}$ b. $3\sqrt[4]{5}$ c. $2\sqrt[5]{6}$ d. $-5\sqrt[3]{4}$ e. Not real
 f. $-3\sqrt[5]{2}$ g. $10\sqrt[3]{3}$ h. $3\sqrt[4]{7}$ i. $6\sqrt[3]{3}$ j. $2\sqrt[5]{10}$
 k. $3\sqrt[3]{3}$ l. $5\sqrt[3]{2}$ m. $-2\sqrt[3]{7}$ n. $2\sqrt[4]{3}$ o. Not real
 p. $4\sqrt[4]{3}$ q. $2\sqrt[5]{4}$ r. $3\sqrt[5]{5}$

“An educational system isn't worth a great deal if it teaches young people how to make a living—but doesn't teach them how to make a life.”

— Unknown